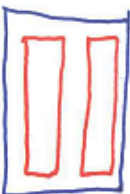
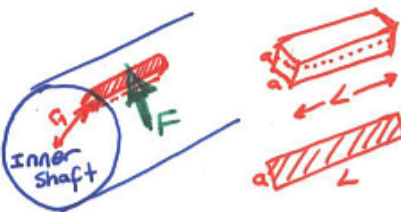


STRENGTH OF MATERIALS I
2nd Midterm Exam
06/12/2013
Solutions

1) $I_{global} = I_{outer} - I_{inner}$ (Caution: centroid remains same)

20 pt  $I_g = \frac{1}{12} \cdot 140 \cdot 200^3 - 2 \cdot \left(\frac{1}{12} \cdot 40 \cdot 160^3 \right)$
 $= 6,603 \times 10^7$

2) 20 pt  $F = \frac{\text{Moment}}{\text{Moment arm}} = \frac{5 \cdot 10^3}{0,1} = 5 \cdot 10^4 \text{ N}$
 $\tau_{max} = \frac{F}{A_{min}} \quad a = \frac{5 \cdot 10^4}{200 \cdot 10^6 \cdot 0,1} = 2,5 \text{ mm}$
 $200 \text{ MPa} = \frac{F}{a \cdot L}$

3) Maximum load the steel rod can withstand

$$P_{max-steel} = \sigma_{max} \cdot A = 200 \cdot 10^6 \cdot \left(\frac{1}{4} \pi \cdot 0,01^2 \right) = 1,571 \cdot 10^4 \text{ N}$$

Remaining load to be supported by aluminum rod = $20 \cdot 10^3 - 15,71 \cdot 10^3 = 4,292 \cdot 10^3 \text{ N}$

Because both rods will elongate same amount:

$$\frac{F_{steel} \cdot L_{st}}{A_{st} \cdot E_{st}} = \frac{F_{al} \cdot L_{al}}{A_{al} \cdot E_{al}} \quad L_{al} = L_{st} \Rightarrow A_{al} = \frac{F_{al} \cdot A_{st} \cdot E_{st}}{F_{st} \cdot E_{al}}$$

30 pt $\frac{1}{4} \pi d_{al}^2 = \frac{4,292 \cdot 10^3 \cdot \frac{1}{4} \pi \cdot 0,01^2 \cdot 209 \cdot 10^9}{1,571 \cdot 10^4 \cdot 69 \cdot 10^9}$

$$d_{al} = 9,097 \text{ mm}$$

$$\sigma_{al} = \frac{4,292 \cdot 10^3}{\frac{1}{4} \pi (0,009097)^2} = 66,03 \text{ MPa} \text{ which is greater than } \sigma_{yield-aluminum}$$

So the reference should be aluminum rod.

$$\frac{F_{st}}{A_{st} E_{st}} = \frac{F_{al}}{A_{al} E_{al}} \quad \text{should be } \sigma_{\text{max-aluminum}}$$

$$\frac{F_{st}}{\frac{1}{4} \pi 0,01^2 209 \cdot 10^9} = \frac{55 \cdot 10^6}{69 \cdot 10^9}$$

$$F_{st} = 13,08 \text{ kN} \rightarrow \sigma_{st} = 166,54 \text{ MPa (safe)}$$

$$F_{al} = 6,92 \text{ kN}$$

$$\frac{1}{4} \pi d_{al}^2 = \frac{6,92 \cdot 10^3 \cdot \left(\frac{1}{4} \pi 0,01^2\right) \cdot 209 \cdot 10^9}{(13,08 \cdot 10^3) (69 \cdot 10^9)}$$

$$d_{al} = \underline{\underline{12,66 \text{ mm}}}$$

4) $I_g = I_{\text{outer}} - I_{\text{inner}}$

$$I_g = \frac{1}{12} 20 \cdot 10^3 - \frac{1}{12} \cdot (20 - 2t) \cdot (10 - 2t)^3$$

$$I_g = 1166,67t - 300 \cdot t^2 + 33,3333t^3 - 1,33333t^4$$

$$M_{ws} = 1,96697 \cdot 10^6$$

$$M_{wh} = 1,63583 \cdot 10^6$$

30px

Because the midship section is symmetric with respect to neutral axis, we consider the max of the 2 moments.

$$M_{\text{max}} = 1,96697 \cdot 10^6$$

$$M_{\text{max}} = \frac{\sigma_{\text{max}} \cdot I}{y} \quad 1,96697 \cdot 10^6 = \frac{200 \cdot 10^6 \cdot I_g}{5}$$

$$\Rightarrow t = \underline{\underline{4,26 \text{ mm}}}$$